Superconductivity-induced conductance suppression in mesoscopic solids

1993 J. Phys.: Condens. Matter 5 L651

(http://iopscience.iop.org/0953-8984/5/50/001)

J. Phys.: Condens. Matter 5 (1993) L651–L662. Printed in the UK

LETTER TO THE EDITOR

Superconductivity-induced conductance suppression in mesoscopic solids

V C Hui † and C J Lambert
School of Physics and Materials, Lancaster University, Lancaster LA1 4YB, UK

Received 27 September 1993, in final form 2 November 1993

Abstract. We examine the change δG in the two-probe electrical conductance G of a mesoscopic sample due to the switching on of superconductivity and prove that when conductance of the normal mesoscopic host is sufficiently high, the onset of superconductivity always produces a decrease in G. If the superconducting order parameter is of magnitude Δ, we focus attention on the susceptibility $\chi_\Delta = \lim_{\Delta\to 0} \partial G(\Delta)/\partial(\Delta^2)$. For weakly disordered, (i.e. ballistic) normal hosts, the average value of this quantity is negative. For diffusive hosts, the mean of χ_Δ increases linearly with $[N - G(0)]$, where N is the number of open channels in the external, current carrying probes, while the fluctuations about the mean are found to be independent of $G(0)$. With increasing normal disorder, χ_Δ undergoes a transition to a region of large fluctuations, before approaching a strongly localized regime, where typical values of χ_Δ are of order $[\ln^2 G(0)]^{-1}$.

When a superconducting island is added to a normal host, one naïvely expects that the electrical conductance of the composite material will increase. While this expectation is borne out by measurements on macroscopic solids, there is no reason to believe that such behaviour can be extrapolated to mesoscopic samples. In this limit, quasi-particles can pass through the system without scattering inelastically and transport properties depend in detail on the quantum interference of propagating waves within a device [1,2]. Recently [3,4,5] it has been demonstrated that mesoscopic samples with superconducting inclusions are attainable experimentally and therefore questions concerning changes in transport coefficients due to the onset of superconductivity, are of immediate interest. The aim of this letter is to demonstrate that, for a sample with sufficiently high conductance, the change δG in the two-probe conductance of a mesoscopic sample, due to the onset of superconductivity can have arbitrary sign and for a sample with a high enough conductance *is guaranteed to be negative*. This result is very general and applies to any structure of size less than the quasi-particle phase breaking length, l_ϕ. The sample may have arbitrary dimensions, be an inhomogeneous mixture or a microfabricated array.

To examine changes in G due to the onset of superconductivity, it is convenient to introduce an appropriate response coefficient. To this end consider a real parameter Δ_0 which characterizes the magnitude of the superconducting order parameter $\Delta(\underline{r})$. For example Δ_0 could be chosen to equal the spatial average of $|\Delta(\boldsymbol{r})|$ over the region occupied by the superconductor. As noted below, to lowest order, δG is of order Δ_0^2 and therefore it is useful to introduce a response coefficient

$$\chi_\Delta = \lim_{\Delta_0\to 0} \frac{\partial G(\Delta_0)}{\partial(\Delta_0^2)} = \lim_{\Delta_0\to 0} \frac{1}{2}\frac{\partial^2 G(\Delta_0)}{\partial \Delta_0^2}.$$

† Permanent address: DRA, Electronics Division, Malvern, Worcestershire WR14 3PS, UK.

This susceptibility, which characterizes the change in G due to the onset of superconductivity, is independent of the magnitude of the superconducting order parameter and will be referred to as the 'Δ susceptibility' of the normal host.

To compute χ_Δ, we start from a formula for the two-probe conductance G, first derived in [6] and which has since been rederived [7, 8] and extended to multi-probe measurements [9]. Consider first a system at zero temperature. For a sample connected to two normal, external, current-carrying leads, if $s^{\alpha,\beta}_{L,L'}$ is the scattering matrix from all incoming β channels (where β refers to particles or holes) of lead L' to all outgoing α channels of lead L and

$$\bar{P}^{\alpha\beta}_{L,L'}(E) = \mathrm{Tr}\{s^{\alpha\beta}_{L,L'}(s^{\alpha\beta}_{L,L'})^\dagger\}$$

then the zero temperature, two-probe electrical conductance, in units of $2e^2/h$, is given by

$$G = T_0 + T_a + \frac{2(R_a R'_a - T_a T'_a)}{R_a + R'_a + T_a + T'_a} \tag{1}$$

where we have taken advantage of particle–hole symmetry at $E = 0$, which yields

$$\bar{P}^{\alpha\beta}_{L,L'}(0) = \bar{P}^{-\alpha\,-\beta}_{L,L'}(0).$$

The coefficients

$$R_0 = \bar{P}^{++}_{L,L}(0) \qquad T_0 = \bar{P}^{++}_{L',L}(0) \qquad \left(\, R_a = \bar{P}^{-+}_{L,L}(0) \qquad T_a = \bar{P}^{-+}_{L',L}(0)\,\right)$$

are probabilities for normal (Andreev) reflection and transmission for quasi-particles from reservoir L, while R'_0, T'_0 (R'_a, T'_a) are corresponding probabilities for quasi-particles from reservoir L'. In the presence of N open channels per lead, these satisfy

$$R_0 + T_0 + R_a + T_a = R'_0 + T'_0 + R'_a + T'_a = N$$

and

$$T_0 + T_a = T'_0 + T'_a.$$

Since the Lambert formula (1) is central to the analysis which follows, it is useful to consider its relationship to the well-known Büttiker formula [10] for normal systems and the Blonder, Tinkham and Klapwijk (BTK) formula for normal–superconducting boundaries [11]. In the normal limit, where all Andreev terms vanish, equation (1) reduces to the Büttiker result [10], $G = T_0$. In the derivation of both the latter and equation (1), the scatterer is well separated from the sources of quasi-particles by normal, current-carrying leads and all scattering amplitudes are obtained by solving the Schrödinger equation or the Bogoliubov–de Gennes equation. In deriving the above result, the chemical potential of the superconductor is determined self-consistently to ensure that on average, quasi-particle charge is conserved. Consequently equation (1) describes situations in which one or more superconducting islands is embedded in a normal background, or in which a superconductor is connected to external sources by normal leads and encompasses experiments such as those of references [3, 4, 5], in which both the superconductors and normal background are phase coherent. The above approach is to be distinguished from that used to derive the BTK formula [11], where the superconductor is treated both as part of the scatterer and as an incoherent source of quasi-particles. In deriving the BTK formula, the chemical potential of the superconductor is imposed externally and on average, the superconductor acts as a sink of quasi-particle charge. It is interesting to note that in the limit of zero transmission, where charge transport due to quasi-particle diffusion is short circuited by Andreev reflection, equation (1) yields for the electrical resistance, $G^{-1} = (2R_a)^{-1} + (2R'_a)^{-1}$. Hence G^{-1} reduces to the sum of two boundary resistances $R_B = 1/2R_a$ and $R'_B = 1/2R'_a$, associated with the left and right leads respectively. The BTK formula for a single boundary resistance at zero temperature is $R_B = 1/(N - R_0 + R_a)$ and therefore in the limit of zero transmission, where $N - R_0 + R_a = 2R_a$, equation (1) reproduces the BTK result. However in situations where quasi-particle transmission is not negligible, the two approaches are distinct. Equation (1) and its finite temperature counterpart must be used when the scattering region is smaller than l_ϕ, while the BTK formula is intended for use when the superconductor is much longer than l_ϕ.

Starting from equation (1), we now prove that if the electrical conductance G of a normal medium is large enough, then the addition of a superconducting region produces a *decrease* in G. More precisely, we show that in the limit $G(0) \to N$, $\chi_\Delta \leqslant 0$. To this end, it is convenient to introduce parameters t_a, r_a which characterize the absence of spatial inversion symmetry and write $T'_a = T_a(1 + t_a)$ and $R'_a = R_a(1 + r_a)$, so that equation (1) yields at zero temperature,

$$G(\Delta_0) = N - R_0 - T_a - \frac{(T_a + R_a)(T_a t_a - R_a r_a)}{(T_a + R_a + R'_a + T'_a)}. \tag{2}$$

Note that in general, t_a, r_a do not vanish when $\Delta_0 \to 0$. From equation (2) one obtains

$$\lim_{\Delta_0 \to 0} \frac{\partial G}{\partial(\Delta_0^2)} = \lim_{\Delta_0 \to 0} -\frac{\partial(R_0 + T_a)}{\partial(\Delta_0^2)} + \eta \tag{3}$$

where

$$\eta = \lim_{\Delta_0 \to 0} \frac{(T_a + R_a)(T_a t_a - R_a r_a)}{(T_a + R_a + R'_a + T'_a)^2} \frac{\partial}{\partial(\Delta_0^2)} (T_a + R_a + R'_a + T'_a). \tag{4}$$

In general, since T_a, R_a, R'_a, T'_a vanish as Δ_0^2 in the limit $\Delta_0 \to 0$, η remains finite in this limit and is of the form

$$\eta = a t_a + b r_a. \tag{5}$$

To prove the theorem, we now take the limit that the normal potential U tends to zero. In this limit, spatial inversion symmetry is restored and consequently $t_a, r_a \to 0$. Hence η vanishes and we obtain

$$\lim_{U \to 0} \chi_\Delta = \lim_{U \to 0} \lim_{\Delta_0 \to 0} -\frac{\partial(R_0 + T_a)}{\partial \Delta_0^2} \leqslant 0 \tag{6}$$

where the last inequality follows from the fact that R_0 and T_a vanish in the limit $U, \Delta_0 \to 0$ and therefore cannot decrease with increasing Δ_0.

The above argument proves the theorem at zero temperature. However since the analysis is separately valid for the contribution to the conductance from quasi-particles of each type and energy, it also applies to the conductance obtained from a thermal average at finite temperatures [6]. On the one hand the theorem appears trivial, since if one starts from a normal system for which G equals an upper bound N, any change δG must satisfy $\delta G \leqslant 0$. On the other, this result is at first sight surprising, because the BTK formula reveals that the current through a perfect normal conductor can be doubled by introducing a superconducting boundary at one end. The essential point is that the two-probe conductance of a phase coherent structure, containing a superconductor of size less than l_ϕ, cannot be viewed as a boundary conductance and equation (1) must be used as a starting point for computing χ_Δ [12].

As a simple example, consider an ideal one-dimensional (1D) system comprising a single delta-function scatterer located at the origin. For such a system, it is convenient to introduce a Fermi wavevector k_F through the relation $\hbar^2 k_F^2/2m = \mu$ and choose potentials $U(x)$, $\Delta(x)$ of the form

$$U(x) = (\hbar^2 k_F/m)U_0\delta(x) \qquad \Delta(x) = (\hbar^2 k_F/m)\Delta_0\delta(x)$$

with Δ_0 real. For such a choice, U_0 and Δ_0 are dimensionless. By matching wavefunctions and their derivatives at $x = 0$, the Bogoliubov–de Gennes equation can be solved exactly. At zero temperature, where only the case $E = 0$ is of interest, particle–hole symmetry is exact and since the system possesses inversion symmetry, the only distinct reflection and transmission coefficients are

$$T_0 = (1 + U_0^2)/d^2 \qquad R_a = T_a = \Delta_0^2/d^2 \qquad R_0 = 1 - T_0 - R_a - T_a$$

where $d = (1 + U_0^2 + \Delta_0^2)$. For this example, $G = 1 - R_0 - T_a = 1/d$ and one finds

$$\frac{\partial G(\Delta_0)}{\partial \Delta_0^2} = -d^{-2} = -G^2.$$

Hence $\partial G(\Delta_0)/\partial \Delta_0^2$ is negative for all (U_0, Δ_0) and vanishes only in the limit $G \to 0$. Although this example is far too simple to model a real physical system, it does illustrate that even when $U_0 = 0$, normal–superconducting–normal systems can possess a finite resistance which increases with increasing Δ_0.

As a second example, consider a perfect 1D system, with $U(x) = \Delta(x) = 0$ for all x, except for the region $0 < x < L$, where $\Delta(x) = \Delta_0$. At $E = 0$, to lowest order in Δ_0, the 'golden rules' for Andreev scattering introduced in references [8, 9] yield

$$R_a = \Delta_0^2 \hbar^{-2} \left| \int_0^L \mathrm{d}x\, |\psi(x)|^2 \right|^2$$

and

$$T_0 = \left| 1 + \Delta_0^2 (i\hbar)^{-1} \int_0^L \mathrm{d}x\, \mathrm{d}x'\, \psi^*(x) G_-^+(x, x', E) \psi(x') \right|^2$$

where

$$\psi(x) = v_F^{-1/2} \exp(\mathrm{i}k_F x)$$

and

$$G_-^+(x, x', E) = (\mathrm{i}\hbar v_F)^{-1} \exp(-\mathrm{i}k_F|x - x'|).$$

Differentiating R_a and T_0 and noting that

$$\frac{\partial}{\partial \Delta_0^2}(R_0 + T_0 + R_a + T_a) = 0$$

yields

$$\chi_\Delta = -(\hbar v_F)^{-2}(\sin^2 k_F L)/(k_F^2).$$

As expected, since the embedding system is clean, this result is always less than or equal to zero.

Having examined two simple examples, we now turn to the more realistic situation of a disordered embedding material in two dimensions, described by a Bogoliubov–de Gennes operator of the form

$$H = \begin{pmatrix} H_0 & \Delta \\ \Delta^* & -H_0 \end{pmatrix}. \tag{7}$$

In this equation H_0 is a nearest-neighbour Anderson model on a square lattice, with off-diagonal hopping elements of magnitude γ and Δ a diagonal matrix with on site, particle–hole couplings of magnitude Δ_0. The scattering region, shown schematically in figure 1, is chosen to be M sites wide and N_s sites long and is connected to external leads of width M. Within the scattering region, diagonal elements $\{\epsilon_i\}$ of H_0 are chosen to be random numbers, uniformly distributed between $\epsilon_0 - W$ and $\epsilon_0 + W$. Within the leads, the diagonal elements of H_0 are equal to a constant ϵ_0. In what follows, for a given realization of the Hamiltonian H, the scattering matrix was obtained numerically, using a transfer matrix technique outlined in appendix 2 of reference [9].

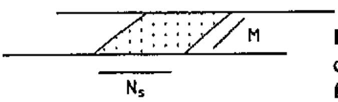

Figure 1. A scatterer of width M sites and length N_s sites, connected to external leads of width M. The current flows from left to right.

Figure 2 shows numerical results for systems of two different lengths $N_s = 30,\ 60$ and two different widths $M = 15,\ 30$. The choice $\gamma = 0.256$, $\epsilon_0 = 4\gamma - 1$ was made and all results are at zero energy. For $M = 30$ this yields a total of $N = 27$ open channels in the external leads. The results of figure 2 were obtained by choosing 3000 values of W, spread uniformly over the interval $0 < W < 2$ and for each value of W, generating a set of random diagonal elements $\{\epsilon_i\}$ for H_0. For each set of diagonal elements, the conductance $G(\Delta_0)$ was first obtained with $\Delta_0 = 0$, then recomputed with $\Delta_0 = 10^{-4}$ and finally the derivative estimated from the difference between the two values. Each point in figure 2 is the result of such a calculation. To aid comparison of results for systems with different widths, all results are shown as functions of the conductance per channel of the normal material, $\tilde{G}(0) = G(0)/N$.

In the limit of a weakly disordered background, with elements ϵ_i of average value zero, a perturbative evaluation of the right hand side of equation (3) reveals that the mean value of χ_Δ, which we denote $\langle\chi_\Delta\rangle$ is of order $\langle\epsilon_i^2\rangle$. Since for a normal system with N open channels, one expects $\langle G(0)\rangle = N - O(\langle\epsilon_i^2\rangle)$, this suggests a linear dependence of the form

$$\langle\chi_\Delta\rangle \simeq -B + C[1 - \tilde{G}(0)] \tag{8}$$

where B and C are positive constants. Figure 2 confirms this mean value relation and clearly demonstrates that as the conductance per channel approaches unity, χ_Δ is always negative. In addition, figure 2 shows that for smaller values of $\tilde{G}(0)$, the change in conductance

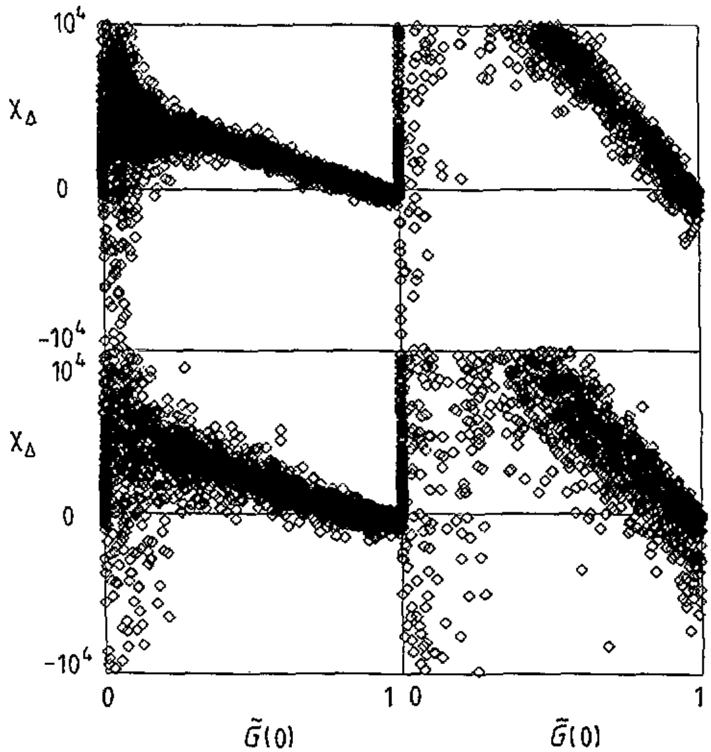

Figure 2. The top left figure shows values of χ_Δ versus the conductance per channel $\tilde{G}(0)$, for system of length $N_s = 30$ and width $M = 30$ sites, obtained from 3000 different realizations of the normal system. The top right figure shows results for a system of size $N_s = 60$, $M = 30$, the bottom left figure for a system of size $N_s = 30$, $M = 15$ and the bottom right for $N_s = 60$, $M = 15$. Values of disorder ranging from $W = 0$ to $W = 2$ were used.

accompanying the onset of superconductivity can have arbitrary sign. We have performed numerical calculations on a range of systems of varying size, which not only confirm equation (8), but also suggest a system size dependence for the constant C of the form $C \sim M^p N_s^{p'}$, with $p \simeq 0$ and $p' \simeq 2$. The behaviour of the fluctuations is somewhat more surprising, appearing to be almost a constant in the regime $1 > \tilde{G}(0) > 0.5$ and then undergoing a transition to large fluctuations for $\tilde{G}(0) < 0.2$. The former behaviour is reminiscent of universal conductance fluctuations [1,2] and suggests that random matrix theory may provide a useful starting point for obtaining more quantitative analytical results in the diffusive regime. Such techniques have recently been used to describe conductance fluctuations at normal–superconducting interfaces [15], but have not yet been applied to susceptibilities such as χ_Δ.

To verify that the region of constant fluctuations in χ_Δ corresponds to diffusive normal hosts, we now present a more detailed analysis of results obtained for a system of size $N_s = 15$, $M = 15$. First we follow the procedure adopted in figure 2 and choose 3000 values of W, spread uniformly over the interval $0 < W < 2$. For each value of W, the conductance per channel $\tilde{G}(\Delta_0)$ of the normal system is shown as a dot in figure 3. Next we choose a discrete set of W values ranging from 0 to 2. For a given W, 500 realizations

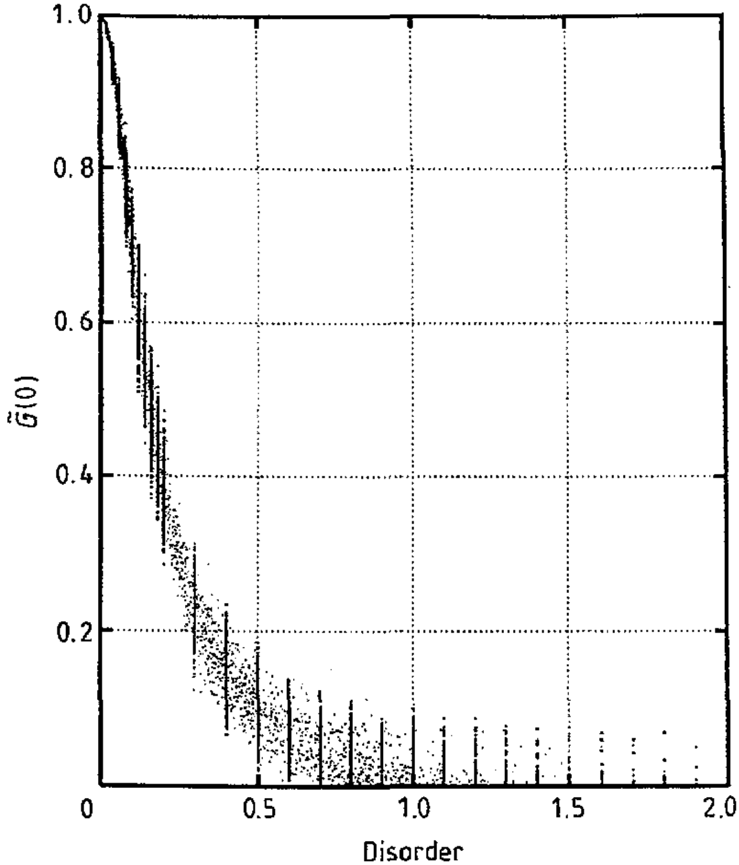

Figure 3. Results for a the conductance per channel $\tilde{G}(0)$ of a normal 15×15 sample. The dots show results obtained using the sampling technique adopted in figure 2. The dense set of points distributed along vertical lines are obtained by choosing a discrete set of values for W and for each W generating 500 realizations of the random site energies.

of the random site energies ϵ_i are generated and the resulting values of $\tilde{G}(0)$ obtained. In figure 3, these form a spread of points along vertical lines at the chosen values of W. From the 500 sets of results obtained for each value of W, we compute the mean $\langle\tilde{G}(0)\rangle$ and standard deviation $\delta\tilde{G}(0)$. These are shown plotted against each other in figure 4, which illustrates that the region of universal conductance fluctuations corresponds approximately to the region $0.2 < \langle\tilde{G}(0)\rangle < 0.7$. For smaller values of $\tilde{G}(0)$, the normal host is localized, while for larger values it is ballistic.

To quantify the behaviour of χ_Δ, for each value of W, 500 results for χ_Δ were also obtained. Figure 5 shows results for the mean $\langle\chi_\Delta\rangle$ and standard deviation $\delta\chi_\Delta$ as a function of $\langle\tilde{G}(0)\rangle$. For weak disorder, where χ_Δ is statistically well behaved, this demonstrates that $\langle\chi_\Delta\rangle$ does not vary monotonically with $1 - \tilde{G}(0)$. Indeed in the ballistic regime, where $\tilde{G}(0) > 0.7$, $\langle\chi_\Delta\rangle$ first decreases with increasing disorder. In the diffusive regime $\langle\chi_\Delta\rangle$ increases monotonically with $1 - \tilde{G}(0)$, while the standard deviation remains constant. Finally for $\langle\tilde{G}(0)\rangle < 0.2$, the distribution of χ_Δ develops a long tail and the mean value is no longer a useful guide to typical behaviour.

To illustrate the cross-over from diffusive to strongly disordered behaviour, figure 6

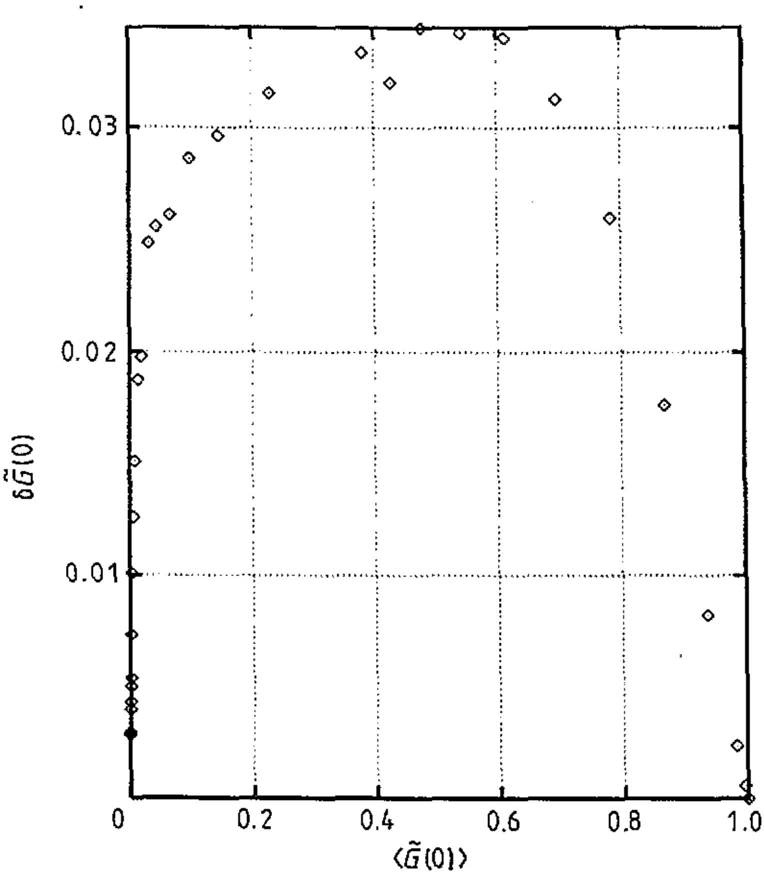

Figure 4. From the 500 results obtained for each of the discrete values of W used in figure 3, the standard deviation $\delta\tilde{G}(0)$ and mean conductance per channel $\langle\tilde{G}(0)\rangle$ can be obtained. This figure shows the resulting plot of $\delta\tilde{G}(0)$ versus $\langle\tilde{G}(0)\rangle$.

shows the same results as figure 5, but with $\tilde{G}(0)$ plotted on a logarithmic scale. Such plots make visible the low-conductance results, which are difficult to discern in figure 5. To illustrate the behaviour at extremely small values of $\tilde{G}(0)$, figure 7 shows the same results as figure 2, plotted as a function of $\log_{10}\tilde{G}(0)$. As $\tilde{G}(0) \to 0$, χ_Δ approaches zero from above, with a negligible probability of negative values.

To obtain a qualitative understanding of this behaviour, consider a highly disordered normal host. In the limit that the fluctuations in the normal potential are much greater than the Fermi energy (measured relative to the bottom of the band), all scattering states $\psi(\underline{r})$ of the normal material decay exponentially into the scattering region [1], with an inverse decay length α and for a scatterer of length L, the conductance $G(0)$ decays as $G(0) \simeq \exp[-2\alpha L]$. For such a system, all transmission coefficients are exponentially small and therefore when $\Delta \neq 0$, electrical conductance is dominated by Andreev reflection at the interfaces with external leads. The dominant contribution to χ_Δ is

$$\lim_{\Delta_0 \to 0} \frac{\partial R_a}{\partial \Delta_0^2}$$

and from the perturbative approach of reference [8], is obtained from integrals of the form

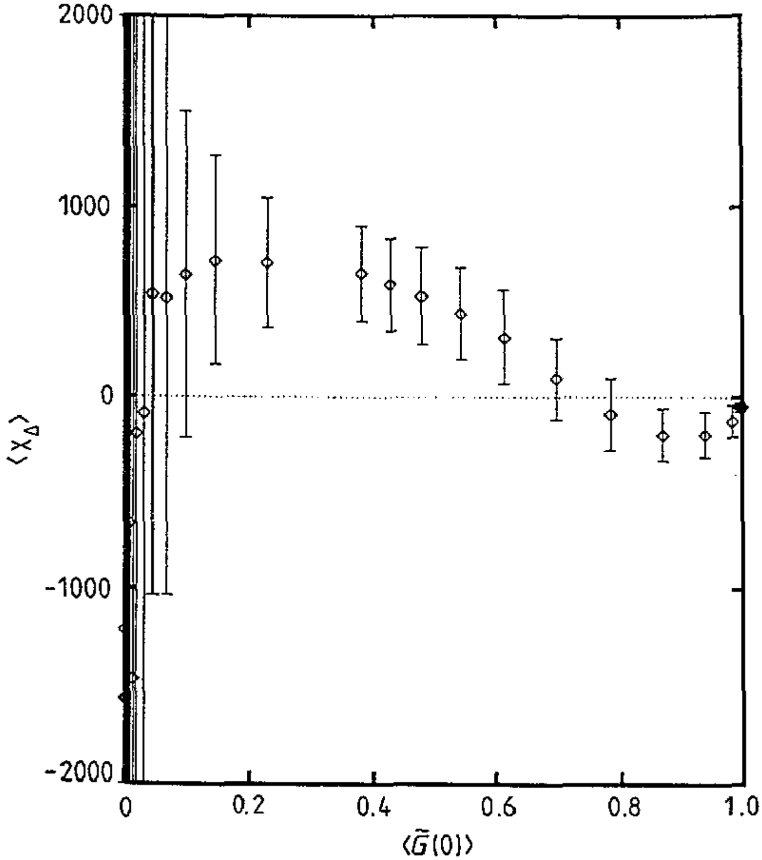

Figure 5. As in figure 4, except that the average $\langle\chi_\Delta\rangle$ is plotted as a function of $\langle\tilde{G}(0)\rangle$, with the standard deviation $\delta\chi_\Delta$ of χ_Δ shown as vertical bars.

$$\left|\int \mathrm{d}^3 r|\psi(\boldsymbol{r})|^2\right|^2 \simeq |\alpha|^{-2}.$$

Hence in the highly localized limit, one expects a typical value of the form

$$\chi_\Delta \simeq A|\alpha|^{-2} \simeq A[\ln G(0)]^{-2} \tag{9}$$

where A is a constant.

After ignoring negative values of χ_Δ, figure 8 shows a log plot of the $N_s = 30$, $M = 30$ results of figure 7. For comparison, the squares show results corresponding to a large potential barrier, with no disorder obtained by choosing $W = 0$ and setting the diagonal elements of H_0 within the scattering region, to an increasingly large constant. The solid lines show plots of the right hand side of equation (9), obtained by adjusting the parameter A to yield a best fit. These indicate that equation (9) correctly predicts the asymptotic behaviour of χ_Δ for a potential barrier, as well as the correct qualitative behaviour for a highly disordered background.

In the presence of disorder there currently exists no quantitative theory for χ_Δ and therefore in this letter we have relied heavily on numerical solutions. Figures 2 and 7

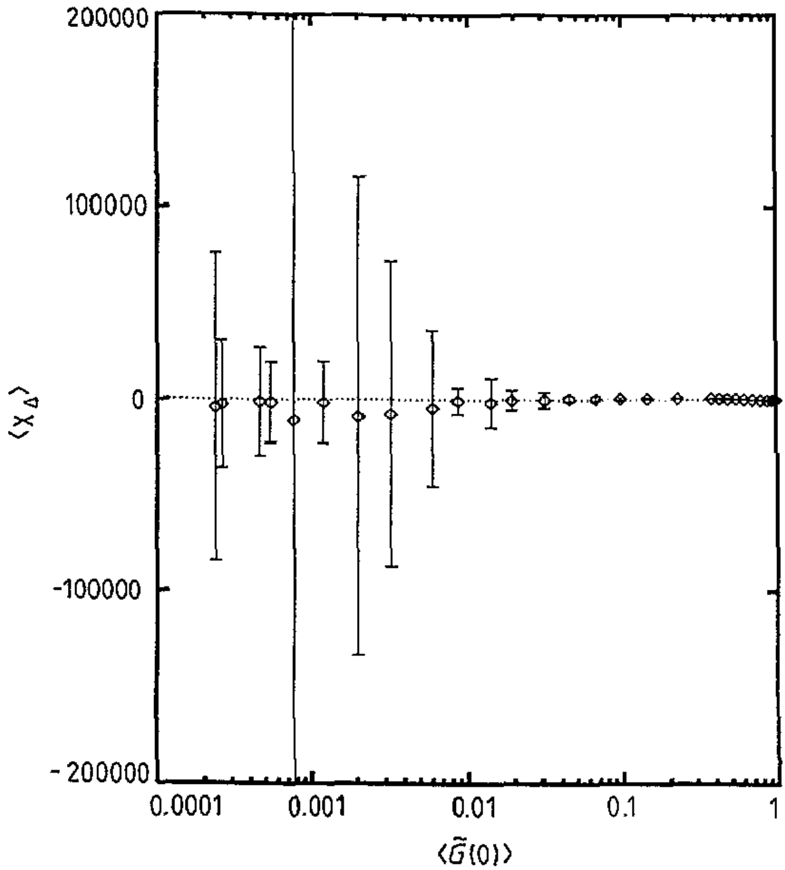

Figure 6. As in figure 5, except that $\langle \tilde{G}(0) \rangle$ is plotted on a logarithmic scale.

suggest that a complete quantitative theory should be capable of describing four distinct regimes, in which χ_Δ exhibits markedly different behaviour. In the ballistic regime, the ensemble average $\langle \chi_\Delta \rangle$ is negative and the theorem proved at the beginning of this letter is exemplified. In the diffusive regime, $\langle \chi_\Delta \rangle$ increases monotonically with $[1 - \tilde{G}(0)]$, whereas the fluctuations are independent of $\tilde{G}(0)$. With increasing disorder, a transition region is encountered in which the fluctuations increase by several orders of magnitude. This regime is particularly interesting, since χ_Δ is very sensitive to small changes in the normal potential and therefore the possibility arises of novel switches and new experimental probes into changes on an atomic scale. Finally in the strongly localized regime, χ_Δ tends to zero, in a manner suggested by equation (9).

In the laboratory, one often considers the response of a physical system to an externally applied field and makes measurements of the appropriate linear response coefficient. In this letter, we have introduced the 'Δ susceptibility',

$$\chi_\Delta = \lim_{\Delta_0 \to 0} \frac{\partial G(\Delta_0)}{\partial(\Delta_0^2)} = \lim_{\Delta_0 \to 0} \frac{1}{2} \frac{\partial^2 G(\Delta_0)}{\partial \Delta_0^2}$$

corresponding to a superconducting order parameter field and have shown that it constitutes a new and non-trivial probe into mesoscopic transport. To date, experimental measurements of

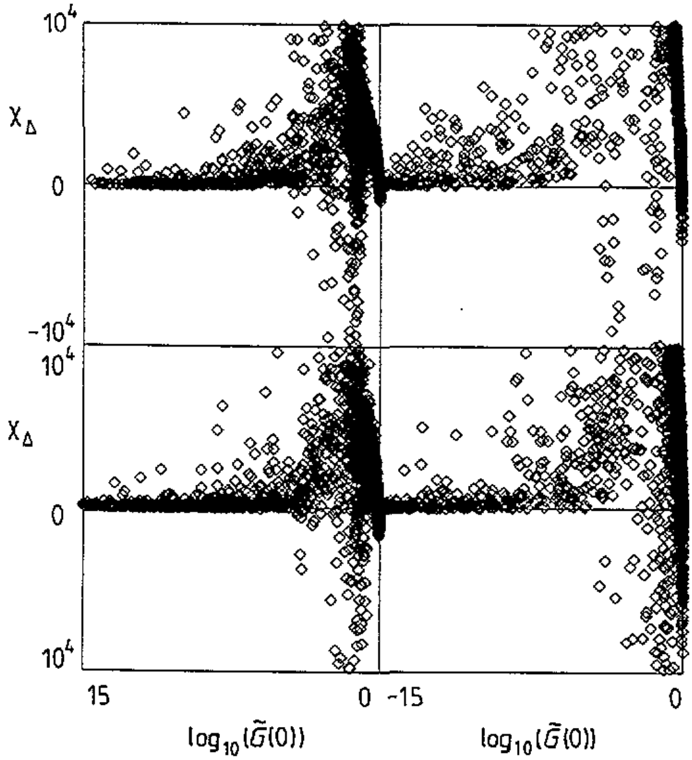

Figure 7. To emphasize the low-conductance behaviour, the results of figure 2, are now plotted against $\log_{10}\tilde{G}(0)$.

resistance increases at the onset of superconductivity [14,15], have been attributed to quasi-particle charge imbalance. The theory presented above presents an alternative mechanism for such anomalies, based on coherent Andreev scattering. The behaviour predicted in this letter is not restricted to temperatures near T_c and persists to zero temperature. The experiments of references [14,15] are complicated by the fact that the external leads, as well as the scattering region, become superconducting below a certain temperature. Consequently as the temperature is lowered, the experiment switches between two distinct physical regimes and the anomaly is destroyed. More clear cut measurements have been reported recently on three micron-sized silver samples with single superconducting islands [16]. Two of these samples yielded negative values for χ_Δ and the third a positive result. Furthermore, in contrast to the to [14,15], the anomaly persists to the lowest temperatures attainable. From the values of χ_Δ obtained above, one notes that typical values of δG can be much larger than e^2/h. This prediction is also borne out by the measurements of reference [16].

This work is supported by the SERC, the EC Human Capital and Mobility Programme and the MOD. It has benefited from useful conversations with S J Robinson, B Kramer, V Petrashov, J-L Pichard and J H Jefferson. Support from the Institute for Scientific Exchange is acknowledged.

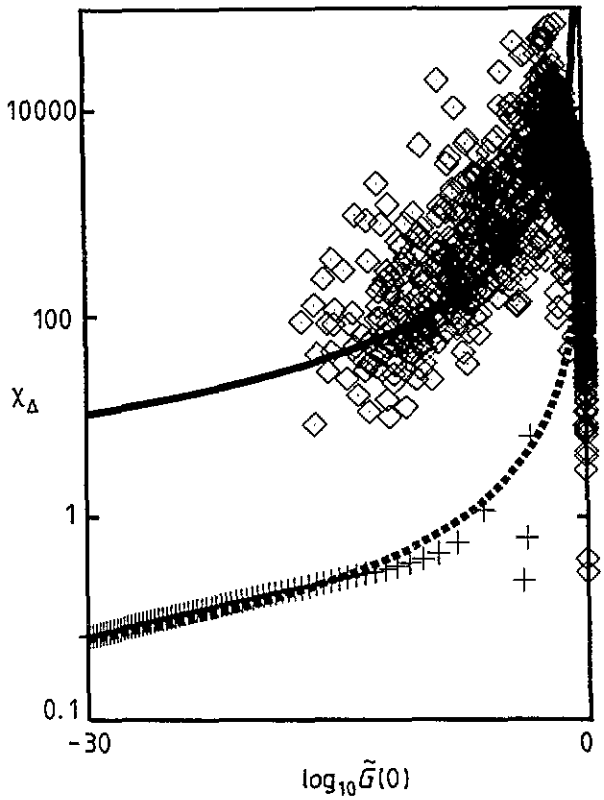

Figure 8. In the limit of an insulating embedding material, this figure shows a log–log plot of values of χ_Δ versus the conductance per channel $\tilde{G}(0)$ of the normal material, obtained from 3000 of realizations of the normal system. The system size is $N_s = 30$, $M = 30$. The diamonds show results for a random potential, while the crosses correspond to an insulating barrier with no disorder. The solid and dashed lines are plots of equation (9), obtained by adjusting the constant A to yield a best fit.

References

[1] Kramer B 1990 *Festkörperprobleme: Adv. Solid State Phys.* **30** 53 (Braunschweig: Vieweg)

[2] For a review of fluctuation phenomena in normal systems see B L Al'tshuler, P A Lee and R A Webb eds 1991 *Mesoscopic Phenomena in Solids* (Amsterdam: North-Holland)

[3] Petrashov V T and Antonov V N 1991 *Sov. Phys.–JETP Lett.* **54** 241

[4] Petrashov V T, Antonov V N and Persson 1992 *Physica Scripta* **T42** 136 (1992),

[5] Petrashov V T, Antonov V N, Delsing P and Claeson T 1993 *Phys. Rev. Lett.* **70** 347

[6] Lambert C J 1991 *J. Phys.: Condens. Matter* **3** 6579

[7] Takane Y and Ebisawa H 1992 *J. Phys. Soc. Japan* **61** 1685

[8] Lambert C J 1993 *J. Phys.: Condens. Matter* **5** 707

[9] Lambert C J, Hui V C and Robinson S J 1993 *J. Phys.: Condens. Matter* **5** 4187

[10] Büttiker M 1988 *IBM J. of Res. and Development* **32** 317

[11] Blonder G E, Tinkham M and Klapwijk T M 1982 *Phys. Rev.* B **25** 4515

[12] Nevertheless one notes that a similar 'theorem' can be proved for the BTK conductance. If the limit $G(\Delta) = 2N$ has been achieved, then any change in Δ must yield $\delta G \leqslant 0$. In practice, perfect Andreev scattering can only be achieved at clean interfaces, in the limit $\Delta/E_F \to 0$, where E_F is the Fermi energy. Therefore increasing Δ will tend to decrease G.

[13] Jalabert R A, Pichard J-L and Beenakker C W J preprint

[14] Kwong Y W, Lin K, Hakonen P J, Isaacson M S and Parpia J M 1991 *Phys. Rev.* B **44** 462

[15] Santhanam P, Chi C C, Wind S J, Brady M J and Bucchignano J J 1991 *Phys. Rev. Lett.* **66** 2254

[16] Petrashov V T 1993 private communication; 1994 *Proc. 20th Int. Conf. on Low Temperature Physics* to be published